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THEORY OF THE HYPERSONIC VISCOUS SHOCK LAYER AT HIGH REYNOLDS NUMBERS AND INTENSIVE INJECTION OF FOREIGN GASES

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E. A. GERSHBEIN
(Moscow)
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The hypersonic flow around smooth blunted bodies in the presence of intensive injection from the surface of these is considered. Using the method of external and internal expansions the asymptotics of the Navier-Stokes equations is constructed for high Reynolds numbers determined by parameters of the oncoming stream and of the injected gas. The flow in the shock layer falls into three characteristic regions. In regions adjacent to the body surface and the shock wave the effects associated with molecular transport are insignificant, while in the intermediate region they predominate. In the derivation of solution in the first two regions the surface of contact discontinuity is substituted for the region of molecular transport (external problem). An analytic solution of the external problem is obtained for small values of parameters $\varepsilon_1 = \rho_\infty / \rho_s^*$ and $\delta = \rho_w^{-1/2} v_w^* / \rho_\infty^{1/2} v_\infty$ in the form of corresponding series expansions in these parameters. Asymptotic formulas are presented for velocity profiles, temperatures, and constituent concentration across the shock layer and, also, the shape of the contact discontinuity and of shock wave separation. The derived solution is compared with numerical solutions obtained by other authors. The flow in the region of molecular transport is defined by equations of the boundary layer with asymptotic conditions at plus and minus infinity, determined by the external solution (internal problem). A numerical solution of the internal problem is obtained taking into consideration multicomponent diffusion and heat exchange. The problem of multicomponent gas flow in the shock layer close to the stagnation line was previously considered in [1] with the use of simplified Navier-Stokes equations.

The supersonic flow of a homogeneous inviscid and non-heat-conducting gas around blunted bodies in the presence of subsonic injection was considered in [2 – 7] using Euler's equations. An analytic solution, based on the classic solution obtained by Hill for a spherical vortex, was derived in [2] for a sphere on the assumption of constant but different densities in the layers between the shock wave and the contact discontinuity and between the latter and the body. Certain results of a numerical solution of the problem of intensive injection at the surface of axisymmetric bodies of various forms, obtained by Godunov's method [3], are presented. Telenin's method was used in [4] for numerical investigation of flow around a sphere; the problem was solved in two formulations: in the first, flow parameters were determined for the whole of the shock layer, while in the second this was done for the sutface of contact discontinuity, which was not known prior to the solution of the problem, with the pressure specified by Newton's formula and flow parameters determined only in the layer of injected gases. The flow with injection over blunted cones was numerically investigated in [5] by the approximate method proposed by Maslen. The flow in the shock layer in the neighborhood of the stagnation line was considered in [6, 8], and intensive injection was investigated by methods of the boundary layer theory in [8 – 12].

1. Investigation of the disintegration of a number of heat-insulating materials [1] had shown that bodies flying in the atmosphere of the Earth and other planets are subjected to a wide range of altitudes and flight velocities, in which the density of the mass of gaseous products of disintegration $(\rho v)_w$ may become equal to or greater than the density of the stream of mass in the oncoming flow $(\rho v)_\infty$. However the ratio of densities ρ_∞ / ρ_w and of velocities v_w / v_∞ remains considerably smaller than unity. Because of this the corresponding ratio of densities of the momentum stream is also much smaller than unity, while the Reynolds numbers $\mathrm{Re} = \rho_\infty v_\infty R / \mu_s^*$ and $\mathrm{Re}_w = \rho_w^* v_w^* R / \mu_w^*$ are considerably greater than unity. We assume that

$$\varepsilon^2 = \mathrm{Re}^{-1} \lessapprox \mathrm{Re}_w^{-1} \ll \delta^2 \lessapprox \varepsilon_1^2$$

The system of Navier-Stokes equations for an N-component chemically reacting mixture of gases in the absence of external electromagnetic fields and energy transport, and under conditions of quasi-inertness, is of the following dimensionless form [13, 14]:

$$\begin{aligned}
&(\rho u r^k)_x + (\rho v r^k a^{-1})_y = 0 \qquad (1.1)\\
&\rho\,(a u u_x + v u_y + a\varkappa u v) + a p_x = \varepsilon^2\,[(\mu u_y)_y + \ldots]\\
&\rho\,(a u v_x + v v_y - a\varkappa u^2) + p_y = \varepsilon^2\,[\ldots]\\
&\rho\,(a u h_x + v h_y) - a u p_x - v p_y = -I_{qy} + \varepsilon^2\,[\mu u_y^2 + \ldots]\\
&\rho\,(a u C_{ix} + v C_{iy}) - \dot{w}_i = -I_{iy} + \cdots \quad (i = 1, \ldots, N)\\
&\sum_{j=1}^{N} A_{ij} I_j = \varepsilon^2 \mu\,[(C_i m)_y + B_i (\ln T)_y + \Gamma_i (\ln P)_y] \quad (i = 1, \ldots, N)\\
&I_q = -\varepsilon^2 \lambda \sigma^{-1} T_y + \sum_{j=1}^{N} h_j I_j + \sum_{j=1}^{N} T_j I_j
\end{aligned}$$

$$p = \rho R_g T \sum_{j=1}^{N} \frac{C_j}{m_j}, \quad \sum_{j=1}^{N} I_j = 0, \quad \sum_{j=1}^{N} C_j = 1, \quad a^{-1} = 1 + \varkappa y, \quad \sigma = \frac{\mu^* c_p^*}{\lambda^*}$$

where xR and yR are coordinates normally attached to the surface of the body, uv_∞ and vv_∞ are velocity components in the direction of these coordinates; rR is the distance of a point in the stream from the axis of symmetry; $\rho_\infty \rho$, $\rho_\infty v_\infty^2\, p$, $v_\infty^2 h$, $(v_\infty^2 / c_p^*)\, T$, $c_p^* c_p$, $\lambda^* \lambda$ and $\mu^* \mu$ are, respectively, the density, pressure, enthalpy, temperature, specific heat, the coefficients of thermal conductivity and of viscosity of the gas mixture; C_i, m_i, $v_\infty^2 h_i$ and $(\rho_\infty v_\infty / R) w_i^{\cdot}$ are, respectively, the concentration, molecular weight, enthalpy and the rate of formation of the i-th component;

$\rho_\infty v_\infty^3 I_q$ and $\rho_\infty v_\infty I_i$ are, respectively, the projections of the thermal and diffusion fluxes of the i-th component on the y-axis. Coefficients A_{ij}, B_i, Γ_i and T_i are functions of temperature, pressure, concentration, charges, molecular weights of components, and binary Schmidt numbers, B_i and T_i depend also on coefficients of thermal diffusion. (These coefficients are readily obtained from [13, 14]). The subscripts ∞, w and $*$ denote dimensional parameters of the oncoming stream, parameters at the surface of the body, and characteristic values of parameters, respectively. Dots denote terms of equations which will not be subsequently required, subscripts x and y denote differentiation with respect to these variables, for plane and axisymmetric flows $k = 0$ and $k = 1$, respectively, and R is a characteristic linear dimension.

Conventional conditions at infinity are specified: $\mathbf{v}_\infty$, ρ_∞, T_∞ and $C_{i\infty}$ $(i = 1, \ldots, N)$.

Boundary conditions at the surface of the body are of the form

$$u = 0, \quad \rho v = G(x), \quad T = T_w(x), \quad I_i + \rho v [C_i - C_i^{(1)}] = \dot\rho_i \quad (i=1,\ldots,N) \qquad (1.2)$$

where $C_i^{(1)}$ is the concentration of the i-th component of the injected gas and $\dot\rho_i$ is the surface rate of formation of the i-th component.

2. For high Reynolds numbers Re and Re_w this problem contains a small parameter at the leading derivative. To solve it we use the method of external and internal expansions.

We assume that all specified functions $G(x)$, $T_w(x)$, $r_w(x)$, $C_i^{(1)}(x)$, etc. are analytic. In that case it is evidently possible to expand the flow parameters outside the mixing region of the oncoming and the injected from the body surface streams, as well as outside the shock wave, into series in integral powers of parameter ε.

The principal terms of the expansion represent the basic inviscid stream, the second terms define the external flow which depends on the thickness of the displaced layer of molecular transport, etc.

Defining the stream function by formula [15]

$$d\psi = \rho u r^k dy - \rho v r^k a^{-1} dx \qquad (2.1)$$

and passing to new variables x and ψ, for the principal terms of the external expansion we obtain the following equations and boundary conditions:

$$\rho u u_x + \rho v v_x + p_x = 0, \quad a v_x - a \varkappa u + r^k p_\psi = 0 \qquad (2.2)$$
$$\rho h_x = p_x, \quad a \rho u C_{ix} = w_i^{\cdot} \quad (i = 1, \ldots, N)$$
$$y_\psi = (\rho u r^k)^{-1}, \quad v = a u y_x, \quad p = \rho R_g T \sum_{i=1}^{N} \frac{C_i}{m_i}$$

At the surface of the body

$$\text{for} \quad \psi = \psi_w \quad \left(\psi_w = -\int_0^x \rho_w v_w r_w^k dx\right)$$
$$y = 0, \; u = 0, \; T = T_w(x), \; G(x)(C_i - C_i^{(1)}) = \rho_i^{\cdot} \quad (i = 1, \ldots, N) \qquad (2.3)$$

Hugoniot formulas apply at the shock wave for $\psi = \psi_s$ $(\psi_s = (k+1)^{-1} r_s^{k+1})$.

The external expansion does not hold in the stream mixing region (region 3 in Fig. 1), but its principal terms satisfy the conditions at the contact discontinuity, where $p_c^+ = p_c^-$ when $\psi = 0$.

Let us attach the system of coordinates x, y to the contact discontinuity and introduce variables defined by formulas

$$y = \varepsilon y^0, \quad v = \varepsilon v^0, \quad I_i = \varepsilon I_i^0, \quad I_q = \varepsilon I_q^0 \tag{2.4}$$

In the region of molecular transport the solution of Eqs. (1.1) expressed in terms of these new variables can now be expanded in integral powers of ε.

For the principal terms of the internal expansion we obtain boundary layer equations whose form is the same as that of equations of system (1.1), if in the latter we omit the dots, set $r = r_c$, $a = 1$, $\varepsilon = 1$, and $\varkappa = 0$, and substitute $p_y = 0$ for the second equation of momenta.

The asymptotic joining of external and internal expansions yields boundary conditions for the principal terms of the internal equation

$$u \to u^{\pm}(x), \quad h \to h^{\pm}(x), \quad C_i \to C_i^{\pm}(x) \quad (i = 1, \ldots, N) \quad \text{for } y^0 \to \pm\infty \tag{2.5}$$

where $u^{\pm}(x)$, $h^{\pm}(x)$ and $C_i^{\pm}(x)$ are, respectively, the velocities, enthalpies, and concentrations of components, which are determined by the external solution at the surface of contact discontinuity.

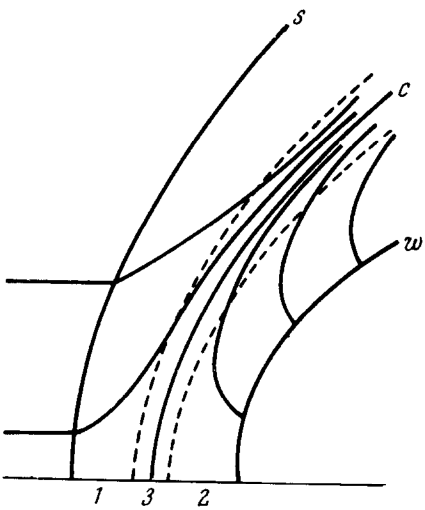

Fig. 1

Equations of the boundary layer with conditions (2.5) determine the structure of the layer of molecular transport (of the suspended boundary layer). The position of that layer is determined by the condition that $v^0 = 0$ for $y^0 = 0$ with an accuracy of the order of ε.

Equations and boundary conditions for subsequent terms of the external and internal expansions are derived as in [16].

3. Let us find the solution of the external problem for small values of parameters $\varepsilon_1 = \rho_\infty / \rho_s^*$ and $\delta = \rho_w^{*1/2} v_w^* / \rho_\infty^{1/2} v_\infty$.

3.1. First, let us consider the flow in region 2 (Fig. 1) between the contact surface and the surface of the body. We omit heuristic considerations and introduce new variables by formulas

$$y = \delta y', \quad u = K^{1/2} u', \quad v = (v_w^* / v_\infty) v', \quad \rho = K^{-1} \rho' \tag{3.1}$$

$$\psi = (K^{-1} v_w^* / v_\infty) \psi', \quad h = K h', \quad K = \rho_\infty / \rho_w^*$$

We express Eqs. (2.2) in terms of these variables and seek its solution in the form

$$\varphi = \varphi_0 + \delta \varphi_1 + \ldots \tag{3.2}$$

where φ is any of the unknown functions. All of these functions and their derivatives are of the order of unity. For the first terms of expansion (3.2) we obtain the following expressions:

$$u_0^2(x, t) = 2\,[h_{0w}(t) - h_0(x, t)], \quad v_0(x, t) = u_0(x, t)\, y_{0x} \tag{3.3}$$

$$y_0(x,\ t) = \frac{1}{r_w^k(x)}\int\limits_t^x \frac{\rho_w(t)\, v_w(t)\, r_w^k(t)}{\rho_0(x,\ t)\, u_0(x,\ t)}\, dt$$

$$C_i^e(x,\ t) = C_{iw}^e \quad (i = 1, \ldots, N_e), \qquad p_0(x,\ t) = p_{0c}(x)$$

$$u_1(x,\ t)\, u_0(x,\ t) = h_{1w}(t) - h_1(x,\ t) \tag{3.4}$$

$$y_1(x,\ t) = -\int\limits_t^x \frac{\rho_w(t)\, v_w(t)\, r_w^k(t)\, dt}{\rho_0(x,\ t)\, u_0(x,\ t)\, r_w^k(x)}\left[\frac{\rho_1}{\rho_0} + \frac{u_1}{u_0} + \frac{k y_0 \cos\alpha}{r_w^k}\right]$$

$$p_1(x,\ t) = \frac{\varkappa}{r_w^k}\int\limits_0^t u_0(x,\ t)\, \rho_w(t)\, v_w(t)\, r_w^k(t)\, dt + p_{1c}(x)$$

where p_{0c} is the pressure at the contact surface, p_{1c} is the correction for pressure p_{0c} which is obtained from the solution of the external problem in layer 1 (Fig. 1) with allowance for (3.3), C_i^e is the concentration of the i-th chemical element, N_e is the number of chemical elements, and t is the value of the x-coordinate of the point of intersection of a streamline and the surface of the body.

In the case of a chemically frozen flow we have

$$C_i(x,\ t) = C_{iw}(t) \quad (i = 1, \ldots, N), \quad \gamma = c_p / c_v, \quad \nu = (\gamma - 1)/\gamma \tag{3.5}$$

$$\frac{\rho_0(x,\ t)}{\rho_{0w}(t)} = \left[\frac{p_{0c}(x)}{p_{0c}(t)}\right]^{1/\gamma_w(t)}, \quad \frac{h_0(x,\ t)}{h_w(t)} = \frac{T_0(x,\ t)}{T_w(t)} = \left[\frac{p_{0c}(x)}{p_{0c}(t)}\right]^{\nu_w(t)}$$

$$\frac{\rho_1(x,\ t)}{\rho_0(x,\ t)} = \frac{1}{\nu_w(t)}\left[\frac{p_1(x,\ t)}{p_0(x)} - \frac{p_{1w}(t)}{p_0(t)}\right] + \frac{\rho_{1w}(t)}{\rho_{0w}(t)} \tag{3.6}$$

$$\frac{h_1(x,\ t)}{h_0(x,\ t)} = \frac{p_1(x,t)}{p_0(x)} - \frac{\rho_1(x,\ t)}{\rho_0(x,\ t)}$$

To simplify integration the specific heats of individual components were assumed independent of temperature.

In the case of flow with balanced chemical reactions the entropy $S(x,\ t) = S_w(t)$. Assuming that h, ρ, T and C_i $(i = 1, \ldots, N)$ are known functions of entropy, pressure, and concentration of elements, we obtain the complete solution of the problem.

It can be readily ascertained that in the case of chemically balanced or frozen flow the systems of equations for subsequent terms of expansion (3.2), may also be integrated in quadratures. Note that in the general case the subsequent terms of that expansion depend on parameter δ, and also on parameter ε_1.

3.2. Let us now consider the flow of gas in region 1 (Fig. 1) between the shock wave and the contact surface. If the form of the contact surface is assumed known, the solution of Eqs. (2.2) in layer 1 can be sought in the form of related series expansions in the small parameter ε_1 [15, 17 – 19]. In this case the flow in layer 1 consists of two sublayers in which the solution is represented by different expansions in ε_1. In the sublayer 1' adjacent to the shock wave the tangent velocity component u or its derivative u_x is of the order of unity, and in the first approximation the flow in that sublayer is defined by Eqs. (2.2) in which terms with lengthwise pressure gradient are absent and the transverse pressure gradient is balanced by centrifugal forces [15]. It should be noted that the solution obtained in [15] is valid in region 1' near the shock wave not only in the neighborhood of the stagnation point of a blunted body, but also at some distance from the latter.

In the sublayer 1" which adjoins the contact surface the flow,is defined in the first approximation by equations of an inviscid boundary layer with allowance for the lengthwise pressure gradient [19].

The asymptotic solution for the whole region 1 may be obtained either by joining the solutions for separate subregions [19] or by constructing and solving asymptotic equations uniformly suitable throughout region 1 [17, 18].

The validity of this approach to the solution of the problem of hypersonic inviscid flow around blunted bodies with impenetrable surface is confirmed by the results of numerical computations [20 – 22]. Similar reasoning was used in [20, 23, 24] for investigating the flow of a hypersonic stream of viscous gas around bodies.

Since the thickness of layer 1 in the case of axisymmetric flow is of the order of $O(\varepsilon_1)$ or of $O(\varepsilon_1 \ln \varepsilon_1)$ in that of plane flow [17], while the thickness of the layer of injected gases 2 is of the order of $O(\delta)$ [1, 12], hence for the shock wave we have

$$r_s(x) = r_w(x)\ [1 + O(\delta) + O(\varepsilon_1) \qquad \text{or} \quad O(\varepsilon_1 \ln \varepsilon_1)]$$

Consequently in the first approximation we obtain $r_{s,0}(x) = r_{c,0}(x) = r_w(x)$.

To find solutions of higher approximations throughout the shock layer it is convenient to use the method of successive approximations, taking into accout the two-layer structure in region 1 and the presence of the layer 2 of injected gases. Application of that method yields for the parameters of flow in the shock layer the following expressions:

$$p_{m,n}(x,t) = \int_{b_m}^{t} \frac{z_{m,n-1}(t)\, a_{m,n-1}}{r^k_{m,n-1}} \left[\varkappa u_m{}^* - \frac{\partial v_m}{\partial x}\right] dt + P_m{}^*(x)$$

$$u^2_{m,n}(x,t) = -2h_{m,n}(x,t) - v^2_{m,n-2}(x,t) + H_m(t)$$

$$y_{m,n}(x,t) = \int_{d_m}^{t} \frac{z_{m,n-1}(t)\, dt}{\rho_{m,n} u_{m,n} r^k_{m,n-1}} + y_m{}^*(x), \qquad \frac{v_{m,n}(x,t)}{a_{m,n-1} u_{m,n}} = \frac{\partial y_{m,n}}{\partial x}$$

$$z_{1,n-1} = \frac{1}{k+1} \frac{\partial}{\partial t} [r_w(t) + y_{s,n-1}(t) \cos\alpha(t)]^{k+1}$$

$$z_{2,n-1} = -\rho_w(t)\, v_w(t)\, r_w{}^k(t), \qquad u_1{}^* = u^0_{1,n}, \qquad u_2{}^* = u_{2,n-1}$$

$$u^{02}_{1,n}(x,t) = u^2_{s,n}(t) + v^2_{s,n-2}(t) - v^2_{1,n-2}(x,t) - \int_t^x \frac{2}{\rho_{1,n-1}} \frac{\partial p_{1,n-1}}{\partial x} dx \qquad (n > 1)$$

$$u^0_{1,1}(x,t) = \cos\alpha(t), \qquad v_1{}^* = v_{1,n-1}, \qquad v_2{}^* = v_{2,n-2}$$

$$a^{-1}_{m,n-1} = 1 + \varkappa y_{m,n-1}$$

$$r_{m,n-1} = r_w + y_{m,n-1}\cos\alpha, \qquad y_{m,0} = v_{m,0} = v_{m,-1} = u_{2,0} = 0$$

$$b_1 = x, \quad b_2 = 0, \quad d_1 = 0, \quad d_2 = x, \quad y_2{}^* = 0, \quad y_1{}^* = y_{c,n}$$

$$p_1{}^* = p_\infty + (1 - \rho^{-1}_{s,n-1}) \sin^2\beta_{n-1}, \qquad p_2{}^* = p_{c,n}(x)$$

$$H_1(t) = u^2_{s,n} + v^2_{s,n-2} + 2h_{s,n}, \quad H_2(t) = v^2_{w,n-2} + 2h_{w,n}$$

$$u_{s,n}\cos(\beta_{n-1} - \alpha) = \cos\beta_{n-1} - v_{s,n-1}\sin(\beta_{n-1} - \alpha)$$

$$h_{s,n} = h_\infty + (1 - \rho^{-2}_{s,n-1})\sin^2\beta_{n-1}, \quad \mathrm{tg}(\beta_{n-1} - \alpha) = a_{s,n-2}\,\partial y_{s,n-1}/\partial x$$

$$\beta_0 = \alpha, \quad \rho^{-1}_{s,0} = \rho^{-2}_{s,0} = y_{s,0} = 0, \quad y_{s,n} = y_{1,n}(x, x), \quad y_{c,n} = y_{2,n}(x, 0)$$

where n is the number of approximation ($n = 1, 2, \ldots$); $m = 1$ for layer 1 and $m = 2$ for layer 2; $y_{c,n}(x)$ and $y_{s,n}(x)$ are the distances from the body to the contact discontinuity and to the shock wave, respectively; α and β are the angles between the direction of the stream at infinity and the tangents to the contour of the body and to the shock wave, respectively, and t is the x-coordinate of the point of intersection between a streamline and the shock wave (when solving the problem in layer 1) or the surface of the body (when solving the problem in layer 2).

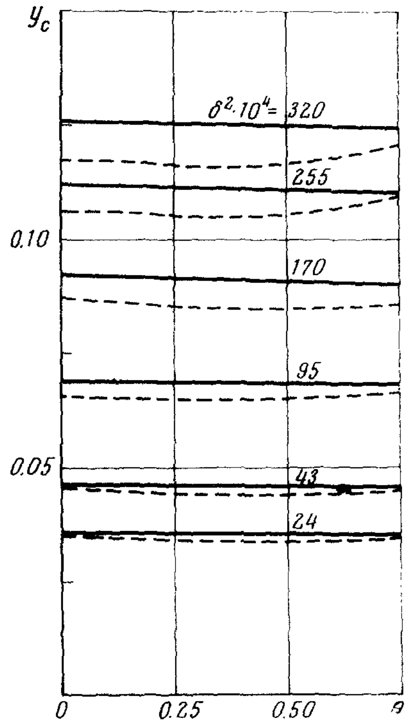

Fig. 2

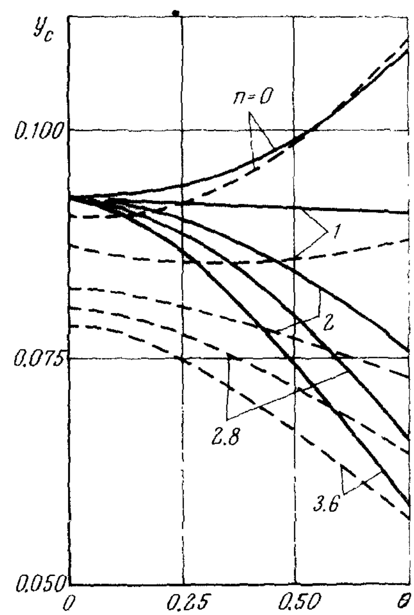

Fig. 3

In the case of flow with frozen chemical reactions in the shock layer and balanced reactions at the surface of the body and at the shock wave we have

$$\frac{h_{m,n}(x,t)}{h_{m,n}(t,t)} = \frac{T_{m,n}(x,t)}{T_{m,n}(t,t)} = \left[\frac{p_{m,n}(x,t)}{p_{m,n}(t,t)}\right]^{\gamma_{m,n}(t)}, \quad \frac{\rho_{m,n}(x,t)}{\rho_{m,n}(t,t)} = \left[\frac{p_{m,n}(x,t)}{p_{m,n}(t,t)}\right]^{1/\gamma_{m,n}(t)}$$

$$C_{im,n}(x,t) = C_{im,n}(t,t) \quad (i = 1, \ldots, N)$$

$$\varphi_{1,n}(t,t) \equiv \varphi_{s,n}(t), \quad \varphi_{2,n}(t,t) \equiv \varphi_{w,n}(t), \quad \varphi = \rho, p, h, T, C_i \ (i = 1, \ldots, N)$$

In the case of flow with balanced chemical reactions $S_{1,n}(x, t) = S_{s,n}(t, t)$ and $S_{2,n}(x, t) = S_{w,n}(t)$, and the enthalpy, density, and concentrations of components depend on $p_{m,n}$, $S_{m,n}$, and the concentrations of elements.

3.3. To estimate the limits of applicability of the asymptotic solution of the external problem we compare it with the numerical solutions presented in [3, 4]. Let us consider the flow of a homogeneous air stream around a sphere with injection of gas distributed according to the law $v_w(\theta) = v_w(0)\cos^n\theta$. We consider the surface temperature T_w to be specified and constant and $\gamma = 1.4$. Pressure distribution along the contact surface, which is unknown before the solution of the problem, will be specified, as in [4], by the Newton formula $p = p_0 \sin^2 \alpha_c$, where α_c is the angle between the tangent to the contact surface and the axis of symmetry. In this case the solution of the problem in the layer of injected gases is separated from the solution in layer 1.

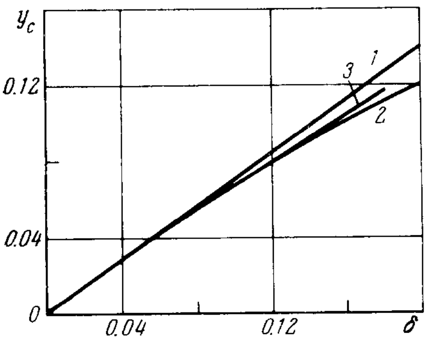

Fig. 4

The thickness of the injected gas layer calculated by formulas (3.3) and (3.5) for the case of injection with $n = 1$ is shown in Fig. 2 for several values of parameter δ^2. The form of the contact surface is shown in Fig. 3 for $\delta^2 = 0.017$ and several values of index n. In these figures the dash lines relate to the numerical solution in [4]. The comparison of the analytic solution in the first approximation (3.3) and (3.5) with the numerical one in [4] shows a satisfactory agreement for $n = 0$; 1. For instance, for

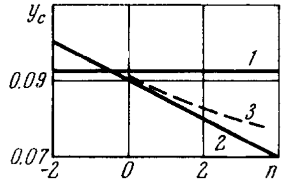

Fig. 5

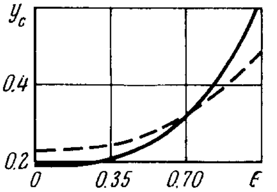

Fig. 6

$n = 1$ the relative discrepancy between the thicknesses is less than 10% even for $\delta = 0.177$. For fixed $\delta^2 = 0.017$ this difference increases with increasing n but does not exceed 17% for $n = 3.6$.

The dependence of the thickness of the injected gas layer on δ for $x = 0$ is shown in Fig. 4, where curve 1 relates to first approximation calculation, curve 2 to that of the first two approximations, and curve 3 is taken from [4]. The similar dependence of the thickness of the injected gas layer on index n is shown in Fig. 5 for $x = 0$ and $\delta^2 = 0{,}017$ (curve 1 relates to the first approximation, curve 2 to two approximations, and curve 3 is taken from [4]). Note that the thickness of layer 2 for $x = 0$ is independent of injection velocity distribution over the surface of the body. Its dependence on this distribution is revealed in the second approximation (3.4) and (3.6).

The comparison of thickness of injected gas layers calculated by formulas (3.3) and

(3.5) and computed numerically in [3] (the dash line) is shown in Fig. 6 for the flow around a spherical blunting for the following parameters:

$$(\rho v)_w / (\rho v_{max})_\infty = 0.5, \quad \gamma = 1.4, \quad p_c = 0.71\,(\rho v^2_{max})_\infty \cdot \cos^2\theta$$

4. The internal problem of multicomponent gas flow in region 3 (Fig. 1) of stream mixing was solved on a computer for a mixture of the following chemical components: O, N, O_2, N_2, H_2, CO, CN, HCN, C_2 and C_3. Reactions in the suspended boundary layer were assumed frozen, and thermal diffusion was neglected. The system of equations for the boundary layer with boundary conditions (2.5) were expressed in terms of the Dorodnitsyn-Lees variables (ξ, η) [10]. It was solved by the implicit four-point two-layer difference scheme [25] with the approximation accuracy $O(\Delta\xi^2) + O(\Delta\eta^4)$.

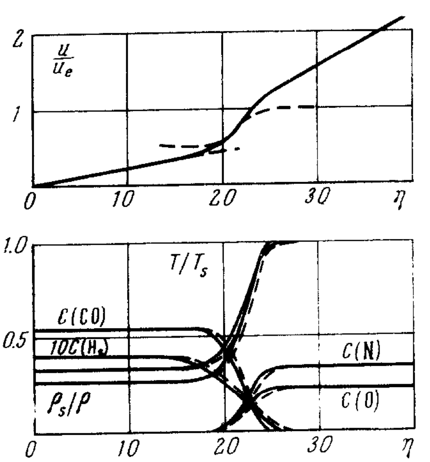

Fig. 7

Some of the calculation results are presented in [12], and are shown here by dash lines in Fig. 7 for the flow in the neighborhood of the stagnation line of an axisymmetric body.

The laminar mixing of homogeneous streams in the presence of a pressure gradient was also considered in [26].

5. Investigations of intensive injection in the theory of the boundary layer [9, 12] show that with increasing injection parameter $\Phi = (k+1)^{1/2} (3/8)^{1/4} \varepsilon_1^{1/4} (\rho_w^* / \rho_s^*)^{1/2}\, \delta\, \mathrm{Re}^{1/2}$ the coefficient of friction, the thermal and diffusion fluxes at the surface of the body decrease and for $\Phi \geqslant 3-5$ the effect of terms which define the molecular transport in equations of the boundary layer on the flow pattern in the immediate vicinity of the body surface becomes negligibly small [10]. This means that for $\Phi = 3-5$ the molecular transport region in the boundary layer begins to move away from the surface of the body, and the flow at the surface is defined by equations of the inviscid boundary layer. The comparison of solutions of inviscid and complete equations of the boundary layer shows a good correlation in a layer of thickness of order $O(\delta)$ close to the surface of the body. A similar flow pattern is obtained by the analysis of the simplified Navier-Stokes equations [1]. A comparison of the numerical solution of the simplified Navier-Stokes equation obtained in [1] with the asymptotic solution (dash lines) is shown in Fig. 7 .

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STABILITY AND BIFURCATION OF COUETTE FLOW IN THE CASE OF A NARROW GAP BETWEEN ROTATING CYLINDERS

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S. N. OVCHINNIKOVA and V. I. IUDOVICH
(Rostov-on-Don)

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Stability and bifurcation of Couette flow between concentric rotating cylinders are investigated for the case when the ratios of their radii R and angular velocities Ω are nearly equal to unity. The limiting problem in the linear theory when $R \to 1$ and $\Omega \to 1$ is the problem of convection stability in the layer [1]. We find that this is also correct in the case of a nonlinear problem. Below we show that solution of the problem of free convection yields the principal term of the expansion of the secondary flow (Taylor vortex) in the powers of a small parameter $\delta = R - 1$. Therefore the results of [2, 3] can be used to provide, in the present case, a strict justification for the use of the Liapunov-Schmidt method to compute the Taylor vortices. The numerical results obtained for the critical Reynolds' number and the amplitude of the secondary flow provide a good illustration of the asymptotic passage as $\delta \to 0$.

1. Statement of the problem. Let a viscous incompressible fluid of unit density fill the space between two infinite concentric cylinders of radii R_1 and R_2, rotating at the angular velocities Ω_1 and Ω_2. Let $R \to 1$ and $\Omega \to 1$, so that

$$(\Omega - 1) / (R - 1) = c = \text{const}, \quad R = R_2 / R_1, \ \Omega = \Omega_2 / \Omega_1$$

We choose $R_2 - R_1$ as the characteristic length and $\Omega_1 (R_2 - R_1)$ as the charac-